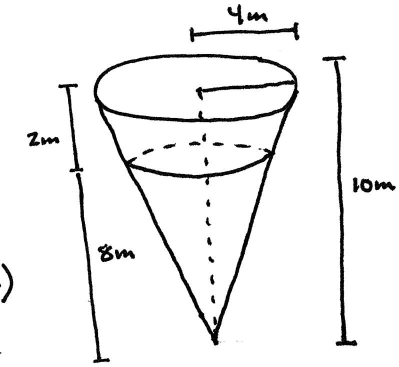
Sect 6.5

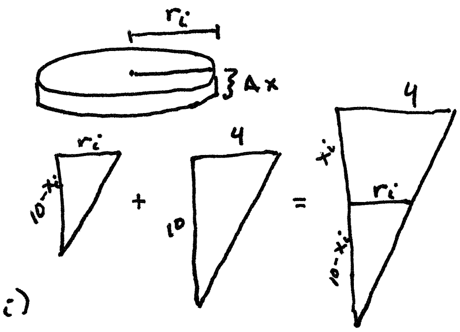
In Class Example



given: We have a tank (see fig). Initially Filled with water to 2m below the top of the tank. Find the work needed to pump all the water out of the tank. (density of $H_2O = 1000\ kg/m^3$)

Solution:

- the water goes from 2m below the top of tank to 10m below the top. So we have the interval $[2, 10]$
- cut the liquid into n layers of water: consider the i^{th} layer
  - the distance each layer must travel to get out of the tank is defined as x_i
  - the layer looks as follows: where r_i is the slice's radius



notice the 2 triangles are similar: so we get:

$$\frac{r_i}{4} = \frac{10-x_i}{10} \Rightarrow r_i = \frac{2}{5}(10-x_i)$$

  - the Volume is $V_i = \pi r_i^2 \Delta x$

so $V_i = \pi\left(\frac{2}{5}(10-x_i)\right)^2 \Delta x = \frac{4\pi}{25}(10-x_i)^2 \Delta x$

  - its mass = density × Volume

$m_i = (1000)\cdot V_i = 160\pi(10-x_i)^2 \Delta x$

  - its Force = mass × gravity

$F_i = m_i \cdot (9.8) = 1568\pi(10-x_i)^2 \Delta x$

therefore, the work acting on each slice is (or layer)

$W_i = F_i d_i = (1568\pi(10-x_i)^2\Delta x)(x_i)$

so $W_i = 1568\pi x_i (10-x_i)^2 \Delta x$

So total work is

$$\lim_{n\to\infty} \sum_{i=1}^{n} 1568\pi x_i (10-x_i)^2 \Delta x$$

$$= \int_2^{10} 1568\pi x(10-x)^2\,dx = 1568\pi \int_2^{10} x(100-20x+x^2)\,dx$$

$$= 1568\pi \int_2^{10} 100x - 20x^2 + x^3\,dx = 1568\pi\left[50x^2 - \frac{20}{3}x^3 + \frac{1}{4}x^4\right]_2^{10}$$

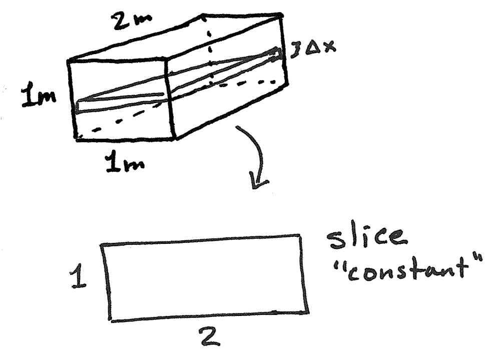
$$= 1568\pi\left[833\tfrac{1}{3} - 150.\tfrac{2}{3}\right] = 3362827.797 \text{ Joules}$$

sect 6.5

exercise 15

given: An aquarium 2 m long, 1 m wide, and 1 m deep is full of water. Find the work needed to pump half of the water out of the aquarium. (density of water = 1000 kg/m³)

Solution:



- the water goes from 0m below the top of the aquarium to $\frac{1}{2}$m below the top. So we have the interval $[0, \frac{1}{2}]$

- we "cut" the water into n layers of water: consider the i^{th} layer.
  - its volume is $V_i = 1m \cdot 2m \cdot \Delta x = 2\,\Delta x \; m^2$
  - its mass = density × volume

    $m_i = d \cdot V_i = (1000 \, kg/m^2)(2\,\Delta x \; m^2)$

    So $m_i = 2000\,\Delta x$ kg
  - its Force = mass × gravity

    $F_i = m_i\, g = (2000\,\Delta x \; kg)(9.8 \; m/s^2)$

    So $F_i = 19600\,\Delta x$ N
  - the distance each layer must travel to get out of the tank is defined by x_i

therefore, the work acting on each layer of water is

$W_i = F_i d_i = (19600\,\Delta x \; N)(x_i \; m)$

So $W_i = 19600\, x_i\, \Delta x$ N-m

So total work is

$\lim_{n\to\infty} \sum_{i=1}^{n} 19600\, x_i\, \Delta x$ Joules

$= \int_0^{1/2} 19600x \, dx$ J

$= 9800x^2 \Big|_0^{1/2} = \frac{9800}{4} - 0 = 2450$ J